

Optimal Reservoir Operation for Irrigation of Multiple Crops Using Genetic Algorithms

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Abstract: This paper presents a genetic algorithm (GA) model for obtaining an optimal operating policy and optimal crop water allocations from an irrigation reservoir. The objective is to maximize the sum of the relative yields from all crops in the irrigated area. The model takes into account reservoir inflow, rainfall on the irrigated area, intraseasonal competition for water among multiple crops, the soil moisture dynamics in each cropped area, the heterogeneous nature of soils, and crop response to the level of irrigation applied. The model is applied to the Malaprabha single-purpose irrigation reservoir in Karnataka State, India. The optimal operating policy obtained using the GA is similar to that obtained by linear programming. This model can be used for optimal utilization of the available water resources of any reservoir system to obtain maximum benefits.

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Introduction

Optimal utilization of irrigation supplies from a reservoir requires knowledge of the reservoir regulation process as well as knowledge of the in-field process at the point of actual water use. Integrating this knowledge to facilitate informed decisions on reservoir operation and cropping pattern generally requires the use of a mathematical model. Reservoir inflow, rainfall on the irrigated area, crop water requirements assessed from potential evapotranspiration, and cropping pattern are the critical inputs for the model.

Dudley and Burt (1973) developed an integrated intraseasonal and interseasonal model for irrigation management using stochastic dynamic programming (SDP) to maximize net benefits from irrigation water for a single crop situation. Bras and Cordova (1981) solved a multistage decision problem using SDP, obtaining optimal temporal allocation of irrigation water. They considered stochastic crop water requirements and the dynamics of soil moisture depletion for a single crop. Vedula and Mujumdar (1992) developed a model to obtain an optimal reservoir operating policy for irrigation of multiple crops with stochastic inflows and crop water requirements by first using dynamic programming to optimally allocate the available water among all crops within a

given period, and then evaluating the system performance using SDP to optimize the benefits over a full year. Vedula and Nagesh Kumar (1996) developed an improved model using a linear programming (LP)-SDP approach considering the soil moisture balance independently for each crop and treating the rainfall in the irrigated area as stochastic for obtaining optimal reservoir operation for irrigation of multiple crops. Optimization analysis of deficit irrigation systems was performed for a sample farm in the Upper Tiber Valley, Italy, by Mannocchi and Mecarelli (1994). Wardlaw and Barnes (1999) have developed an optimization approach for optimal allocation of irrigation water supplies in real time and demonstrated its applicability to a run-of-river system. Paul et al. (2000) developed an optimal resource allocation model that optimized irrigation water allocation and areas of cultivation for the cropping pattern considered. First the optimal seasonal allocation of water and optimal cropping pattern for maximizing the net benefits are determined. The results are then used for a single crop intraseasonal model (SDP) giving optimal weekly irrigation allocations for each crop.

Application of genetic algorithms (GA) for irrigation planning is relatively new. Wardlaw and Sharif (1999) evaluated the performance of GA for a four-reservoir problem. Sharif and Wardlaw (2000) presented a GA approach to the optimization of a multireservoir system. Results of the GA compared well with those obtained by discrete differential dynamic programming. Raju and Nagesh Kumar (2004) applied a GA for evolving an optimum cropping pattern utilizing surface water resources in the command area of a multipurpose reservoir system. Morshed and Kaluarachchi (2000) employed three GA enhancement methods to a nonlinear groundwater problem for minimizing the costs of pumping for meeting a specific demand. Hilton and Culver (2000) compared an additive penalty method with a multiplicative penalty method in a GA for minimizing the cost of groundwater remediation. Reed et al. (2000) presented a review of the existing tools from literature to ensure that a GA converges to an optimal or near-optimal solution. Wu and Simpson (2001) applied a messy GA to the optimal design of a water distribution

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system requiring fewer design trials than for other GAs. Yoon and Shoemaker (2001) applied a real-coded genetic algorithm with directive recombination and screened replacement, for in situ bioremediation of groundwater.

Although GA is convincingly adopted for many other optimization problems, it was seldom applied to an irrigation allocation problem. It is therefore proposed in this study to adopt GA for optimization of irrigation allocation for multiple crops. Results from this model will be compared with corresponding results from LP optimization model for the same problem.

Genetic Algorithms

GAs are combinatorial optimization methods that search for solutions using an analogy between optimization and natural selection. The methodology of GAs involves coding, fitness function computation, and operations of reproduction, crossover, and mutation (Goldberg 2000). The advantages of GAs are that they (1) work with coding of the parameter set but not with the parameters themselves, (2) search from a population of points, not a single point, (3) use objective function information itself but not any derivatives, and (4) use probabilistic transitions rules but not deterministic rules.

A constrained problem is converted into unconstrained problem in a GA by introducing a penalty function as follows:

$$F_i = f(x) + \epsilon \sum_{j=1}^k \delta_j \langle \phi_j \rangle^2 \quad (1)$$

where F_i =fitness value; $f(x)$ =objective function value; k =number of constraints; $\epsilon=-1$ for maximization and $+1$ for minimization; δ_j =penalty coefficient; and $\langle \phi_j \rangle$ =amount of violation.

Irrigation Allocation Model

In this paper a GA based reservoir operation model is formulated to allocate the water available for each season optimally between different crops for each time period of different growth stages. The objective is to maximize the sum of the relative yields of all crops, given inputs of reservoir storage at the beginning and end of the season, inflow, rainfall on the irrigated area and crop water requirements assessed from potential evapotranspiration. The model also takes into account the intraseasonal competition for water among multiple crops, soil moisture dynamics for each cropped area, and the heterogeneous nature of the soil and crop response to the level of irrigation applied. For the present study, in the case of the LP model, the following assumptions are essential: (1) Crop root growth is linear; (2) reservoir elevation versus storage curve is linear; and (3) the relation between the ratio of the actual evapotranspiration (AET) to the potential evapotranspiration (PET) and the corresponding soil moisture content is linear. All these assumptions are essential for LP but are not required in a GA. However, these assumptions are followed for the GA also, to make a comparison between the GA and LP models possible.

Objective Function

The objective function for allocation of water among various crops is developed to maximize the sum of relative crop yield for the specified cropping pattern:

$$Z = \text{Max} \sum_{c=1}^{NC} \left\{ 1 - \sum_{g=1}^{NGS} ky_g^c \left(1 - \frac{\sum_{t \in g} AET_t^c}{\sum_{t \in g} PET_t^c} \right) \right\} \quad (2)$$

where Z =sum of relative yields of all crops; c =crop index; NC =number of crops; NGS =number of growth stages; ky_g^c =yield response factor for the growth stage g of the crop c ; AET_t^c =actual evapotranspiration for period t for crop c (depth units); and PET_t^c =potential evapotranspiration for period t for crop c (depth units).

The summation of AET and PET is for the periods within the growth stage g for crop c . The maximum value of the objective function will be 1 when the allocation of available water is such that $AET=PET$ for each crop in each period. Irrigation water allocation is made to ensure that soil moisture in the root zone is above the permanent wilting point and below the field capacity. The model computes the irrigation required to bring the soil moisture in the root zone to the field capacity in each time step of ten days. Continuous irrigation is contemplated without any rotational allocation for various crops.

Reservoir Water Balance

The reservoir water balance is governed by the storage continuity equation (Loucks et al. 1981):

$$(1 - a_t)S_t + Q_t - R_t - OVF_t - A_0 e_t = (1 + a_t)S_{t+1}, \quad \forall t \quad (3)$$

where S_t =active storage at the beginning of the period t ; Q_t =inflow during the period t ; R_t =release for the period t (for irrigation); OVF_t =overflow for the period t ; A_0 =water spread area at dead storage level; A_a spread area per unit volume of active storage; and e_t =evaporation rate in period t , and

$$a_t = A_a e_t / 2, \quad \forall t \quad (4)$$

In the previous equation S_t , Q_t , R_t , and OVF_t are in units of million cubic meter ($\times 10^6 \text{ m}^3$) and e_t is in units of millimeters. X_t , the total amount of irrigation water made available at the farm level, is given by

$$X_t = \eta R_t \quad (5)$$

where η =conveyance efficiency.

Irrigation release, R_t , is subject to the constraint of the carrying capacity of the canal. Reservoir storage in any period should not exceed its active storage capacity, S_{\max} :

$$S_t \leq S_{\max}, \quad \forall t \quad (6)$$

Soil Moisture Balance

The change in soil moisture in any time period is governed by the soil moisture balance equation incorporating the increase in root depth during the period. At the beginning of the season, there will be no crops and so the rainfall normally occurring prior to the season is assumed to ensure that the soil moisture is at the field capacity.

$$SM_1^c = SM_{\max}^c, \quad \forall c \quad (7)$$

where SM_1^c =soil moisture above the permanent wilting point at the beginning of the first period ($t=1$) for the crop c and SM_{\max}^c =soil moisture at the field capacity for the crop c . It is assumed that the soil moisture is at the field capacity in the incremental depth over which the crop root grows during each period. The soil moisture is expressed in depth units per unit root depth of the crop. The soil moisture balance equation for a given crop c for any period t is given by

$$SM_{t+1}^c D_{t+1}^c = SM_t^c D_t^c + IR_t + x_t^c - AET_t^c + SM_{\max}^c (D_{t+1}^c - D_t^c) - DP_t^c, \quad \forall c, t \quad (8a)$$

where SM_t^c =available soil moisture at the beginning of the period t for the crop c ; D_t^c =average root depth of crop c in period t ; IR_t =rainfall over irrigated area in period t (depth units); x_t^c =irrigation water allocated to crop c in period t (depth units); and DP_t^c =deep percolation during the period t for crop c . Deep percolation (depth units), if any, can be computed as follows:

$$DP_t^c = [SM_t^c D_t^c + IR_t + x_t^c - AET_t^c] - SM_{\max}^c D_t^c, \quad \forall c, t \quad (8b)$$

The available soil moisture in any period t for crop c cannot exceed the field capacity

$$SM_t^c \leq SM_{\max}^c, \quad \forall c, t \quad (9)$$

In general, AET remains the same as PET when the soil moisture is at field capacity and also for a small fractional reduction from the field capacity. However, to make the problem amenable for solution by LP, the reduction in soil moisture is assumed to be uniform right from the field capacity to the permanent wilting point as done in earlier studies (Vedula and Nagesh Kumar 1996). The linear relationship between AET, PET, and the soil moisture (between permanent wilting point and field capacity) is

$$AET_t^c \leq \frac{SM_t^c D_t^c + IR_t + x_t^c}{SM_{\max}^c D_t^c} PET_t^c, \quad \forall c, t \quad (10)$$

The upper bound for AET is PET:

$$AET_t^c \leq PET_t^c, \quad \forall c, t \quad (11)$$

The heterogeneous nature of soils within the irrigated area can be taken into account by modeling the soil moisture balance for each crop and soil type individually, i.e., Eqs. (7)–(11) will be adapted for each soil type.

Allocation Constraints

Irrigation water within a growth stage is provided uniformly among all the time periods of that growth stage to avoid undue concentrations (Ashok 2002):

$$x_t^c = \frac{RG_g^c}{NP_g}, \quad \forall c, t \quad \text{except for } t \text{ belonging to } g = 1 \quad (12)$$

where RG_g^c =irrigation allocation for the growth stage g of the crop c (depth units); and NP_g =number of time periods in the growth stage g .

As the soil moisture is assumed to be at the field capacity at the beginning of the first time period, irrigation requirements will be nil during that period. In any period the total water allocated to all crops should be within the water available for allocation, X_t .

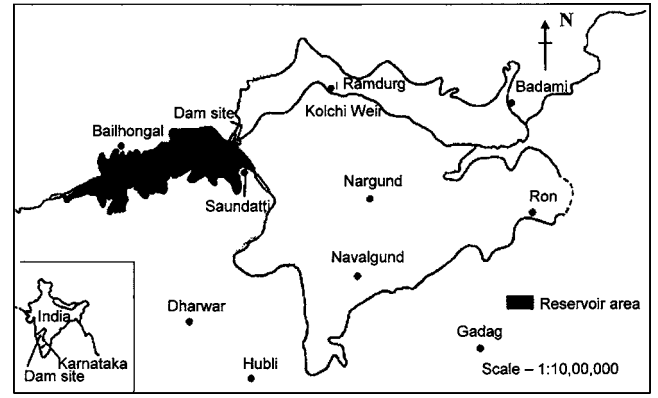


Fig. 1. Location map of Malaprabha Reservoir

$$\sum_c x_t^c \text{AREA}^c \leq X_t, \quad \forall t \quad (13)$$

where AREA^c =area irrigated under crop c . For any growth stage of any crop, the total allocation made should be equal to the sum of allocations made in all the periods of that growth stage

$$\sum_{t \in g} x_t^c = RG_g^c, \quad \forall c, g \quad (14)$$

Irrigation water is not allocated to a crop for any period t , which lies outside the growing season of that crop. This constraint is relevant, because all crops may not start at the same time and may not have the same duration.

The objective function given in Eq. (2) and the constraints in Eqs. (3)–(14) constitute the GA model which is implemented adopting binary code for the decision variables. The decision variable for the problem is x_t^c , irrigation water allocated to crop c in period t in depth units. Binary string length for the decision variable, x_t^c , is taken as 10 bits (decided based on the range of values for the variable) separately for each crop for each time period. For example, in kharif season, when there are three crops and 15 periods, with a string length 10 bits each, there will be a total string length of 450 bits. S_t and SM_t^c are calculated for each value of x_t^c considering the inflow, rainfall in the irrigated area, and potential evapotranspiration. It may be noted that the continuity equations for S_t and SM_t^c [Eqs. (3) and (8)] will be satisfied due to the penalty function imposed on the continuity constraints of the model.

Case Study

The model has been applied to the right canal command area of the Malaprabha single-purpose irrigation reservoir in Karnataka State, India. The project is located at latitude $15^\circ 49' N$ and longitude $75^\circ 6' E$. The catchment area of the river up to the dam site is 2,564 km². The area of the reservoir at full reservoir level is 13,578 ha. The reservoir has gross and live storage capacities of 1,070 and 870×10^6 m³. The mean annual inflow is $1,348.61 \times 10^6$ m³. The mean annual rainfall in the command area is 576 mm. Fig. 1 shows the location map of the Malaprabha reservoir. There are two main canals under this project. The left bank canal serves a command area of 53,137 ha and the right bank canal serves 1,286.34 ha. As the left bank canal command is not fully developed irrigation was restricted to the right bank

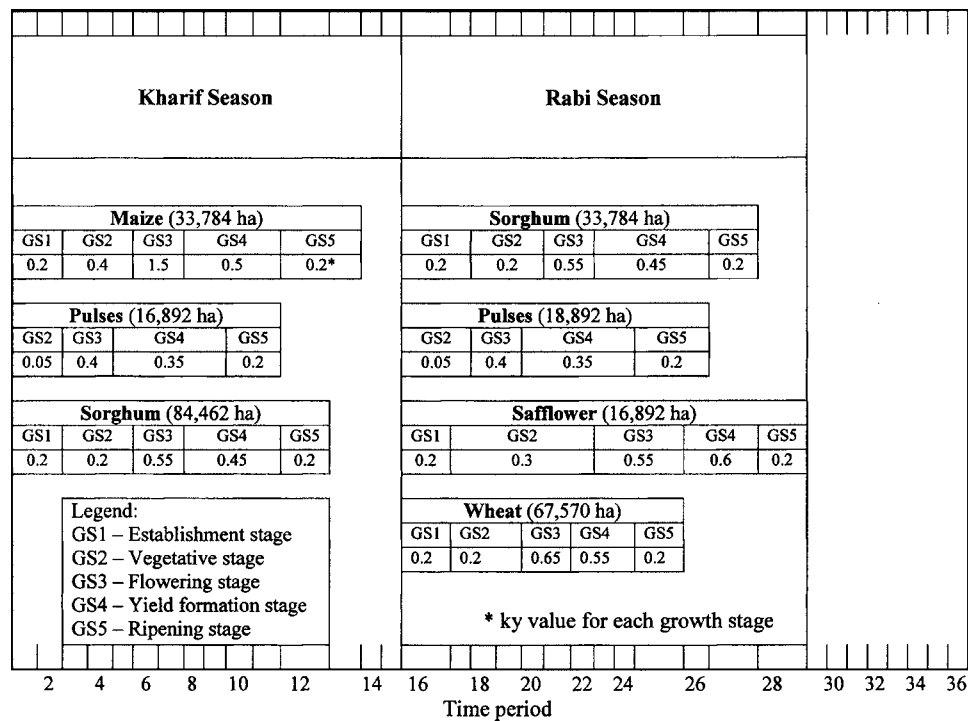


Fig. 2. Crop calendar adopted in the command area

canal only. So the developed model is applied only to the right bank canal command. For this reservoir there are no mandatory downstream requirements.

The soil in a major portion of this command area is black cotton soil (Montmorillonite, categorized as CH as per the unified soil classification system) and hence this soil type only was considered as was done in earlier studies (Vedula and Mujumdar 1992; Vedula and Nagesh Kumar 1996). However, it may be noted that this is not a limitation of the model, as it can handle multiple soil types by considering each soil type individually by adapting Eqs. (7)–(11) for each soil type.

The farmers in this command area (with small holdings of less than 20 ha) are traditional and adopt a very similar cropping pattern every year. They do so to meet their own food requirements and the decision is not commercially driven. Therefore the cropping pattern being adopted in the field, which is recurrent, is used in this model. Total crop water requirements used in this model were computed based on potential evapotranspiration. These requirements for the same area were evaluated earlier in previous studies by Vedula and Mujumdar (1992) and Vedula and Nagesh Kumar (1996).

Results and Discussion

The water year (June–May), is divided into 36 periods. There are two cropping seasons: kharif (Periods 1–15) and rabi (Periods 16–31). The last 5 periods of the year, that is, Periods 31–36, will have no irrigation activity. The crops grown in the kharif season are sorghum, maize, and pulses and in rabi season wheat, sorghum, safflower, and pulses. The existing cropping pattern in the irrigated area showing principal crops, crop calendar, and the area irrigated during each season is presented in Fig. 2.

In addition, duration of each growth stage for each crop and the corresponding yield response factors (indicated in parentheses following each growth stage) are also shown therein. For this cropping pattern, the proposed model optimizes the allocation of the available water for each period from the reservoir. The values of A_0 and A_a in the reservoir continuity equation [Eq. (3)] are $37.01 \times 10^6 \text{ m}^2$ and $0.117 \times 10^6 \text{ m}^2/\text{m}^3$, respectively.

The model was run with a crossover probability of 0.8, a mutation probability of 0.05 and a population size of 10. The maximum number of generations is fixed as 20. The GA model is run for different inflow and rainfall states at the beginning of the season. For this purpose, seasonal inflow and rainfall variables are discretized into five discrete states based on statistical analysis of the available data (State 1 representing the lowest value and State 5 representing the highest value). These states represent various stages of the reservoir storage at the beginning of the season and the seasonal rainfall in the irrigated area. These states can as well represent various probabilities of exceedance of the respective variables. Seasonal values are disaggregated into corresponding values for ten day periods based on conditional expectations estimated using the historic data, i.e., conditional probability of the ten day value in a particular class interval, given that the seasonal value is in a specified state, is computed from the historic data for different class intervals and its expected value is then determined (Vedula and Nagesh Kumar 1996).

As indicated in the objective function and the subsequent explanation, when sufficient water is available, it is allocated to all the crops to meet the crop water requirements. When there is deficit supply, weightage is given to allocate irrigation water to the crops which are more sensitive to the deficit condition. Weightages are decided based on the yield response factor (ky) for each crop for each growth stage given in Fig. 2. For example, when there is a deficit supply during Time Period 6 of the kharif

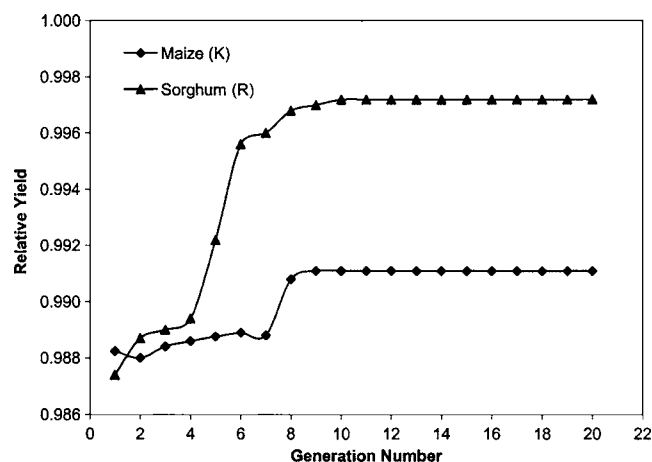


Fig. 3. Relative yield of maize in kharif season and sorghum in rabi season for different generations of GA for State 5

season, first priority will be given to maize (with k_y of 1.5) and next to sorghum ($k_y=0.55$) and last to pulses ($k_y=0.35$) in the proportion of k_y values. It may be noted that such apportioning is done distributing deficits over the entire crop season using the optimization model.

Fig. 3 presents variation of the relative yield of the maize crop in the kharif season and sorghum crop in rabi season (when the inflow and rainfall were highest, i.e., in Discrete State 5) for different GA generations. GA model is solved for the remaining four discrete states of seasonal inflow and seasonal rainfall. For lower values seasonal inflow and seasonal rainfall states the yields are much lower and there is more stress for irrigation supplies. Optimal ten day releases for irrigation of each crop thus obtained for various discrete states of seasonal inflow and seasonal rainfall constitute the derived operation policy for the reservoir. However, results corresponding to Discrete State 5 only are presented for illustration.

AET values obtained with the GA allocation model are compared with those of a LP model in Fig. 4 for maize, pulses, and sorghum in kharif season. It is observed from Fig. 4 that AET values obtained by LP for maize are higher for irrigation allocation Periods 6, 8, and 10 whereas the values are practically the same for the remaining periods. Fig. 5 presents similar comparison of AET values for sorghum, wheat, pulses, and safflower in rabi season. Significant differences are observed in Periods 3 and 7 for sorghum; 4 and 8 for pulses; 3 and 5 for wheat; and 2 for safflower.

It is observed from Figs. 4 and 5 that AET values obtained by the GA and the LP compared well for most of the periods. The computational time required for the GA is practically insignificant compared to the time required for LP. For example for the kharif season, the computational time required for the GA is 50 s whereas the LP required 180 s on a Pentium III computer. Moreover the number of generations in the GA (in this case 20) is not comparable to the number of iterations in the LP (around 450 for kharif season). In addition all the assumptions required for the LP model (stated earlier) are not required for the GA thus rendering the GA more realistic.

From this study, it is apparent that the GA performs well and is efficient when compared with LP. However, the LP model contains a very simple optimization approach due to the required but unrealistic simplification hypotheses of linearity (e.g., the relation between AET/PET and the corresponding soil moisture

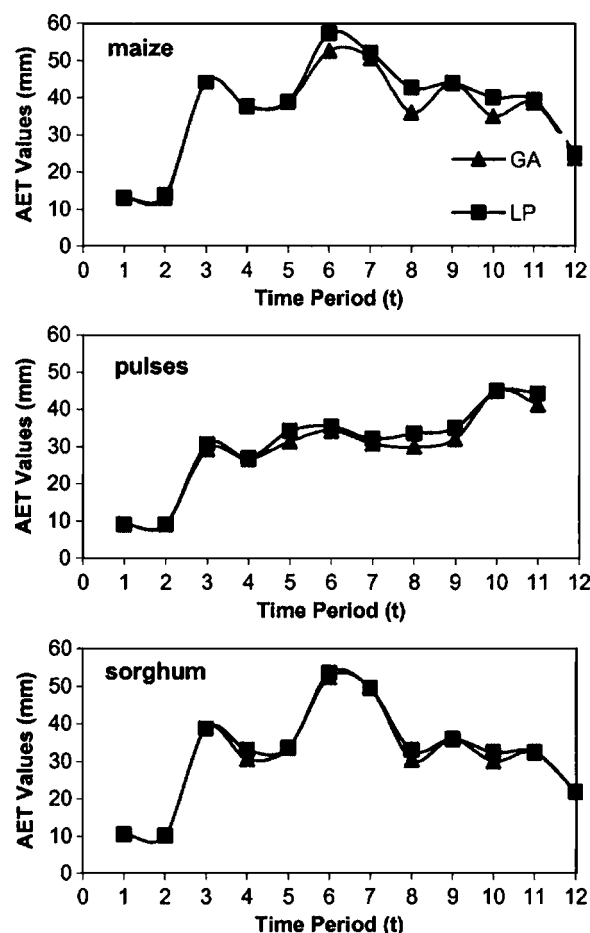


Fig. 4. Comparison of AET values by GA and LP for kharif season

content is linear). The GA model proposed in this study can be further improved by incorporating nonlinear constraints to overcome the simplifications inherent in the LP model.

It is relevant to note that only one soil type was considered in the case study representing the predominant soil type in the irrigated area. However, in general, more than one soil type will be prevailing. As already indicated in the section on case study, this model can handle multiple soil types by adopting Eqs. (7)–(11) for each soil type.

Conclusions

An irrigation allocation model was developed to optimize relative yield from a specified cropping pattern for various states of reservoir inflows and rainfall in the irrigated area. The model integrates the reservoir releases with the consequent soil moisture at the root level for each crop and for each period. The model was applied to an existing single purpose reservoir in Karnataka State, India. In rabi season AET obtained is almost equal to PET indicating that optimal allocations were obtained from this model. It is observed that AET values obtained by GA and LP compared well. The operating policies evolved by the study can be adopted in the field, for optimizing the utilization of the existing resources to obtain maximum benefits.

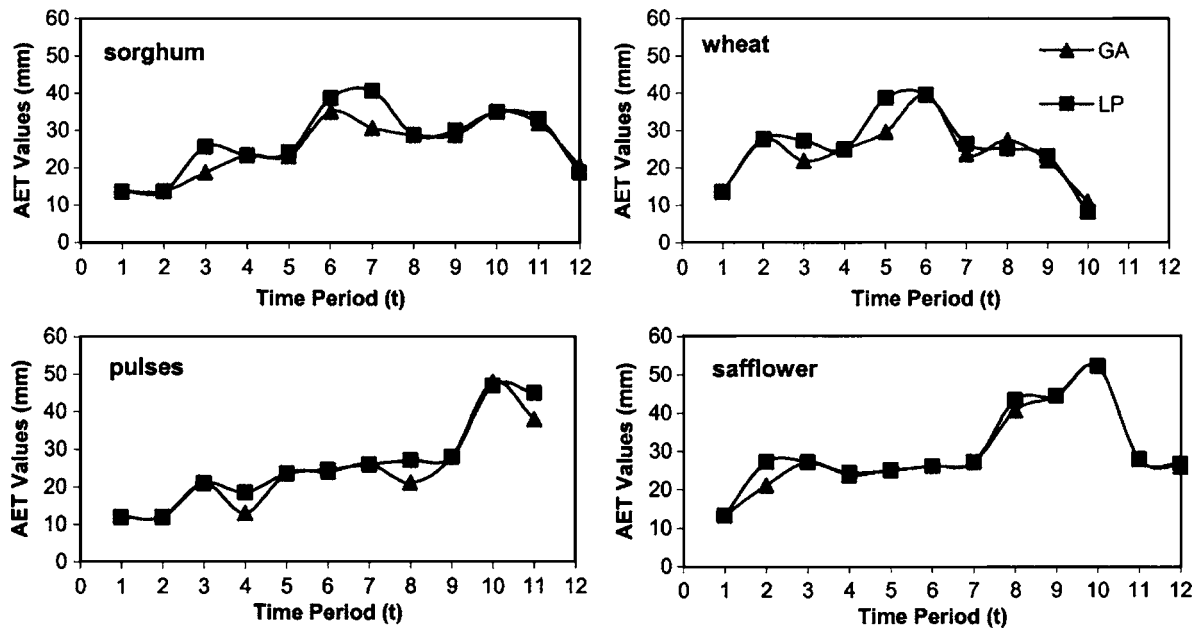


Fig. 5. Comparison of AET values by GA and LP for rabi season

Notation

The following symbols are used in this paper:

- A_a = water spread area per unit active storage volume;
- A_0 = water spread area at dead storage level;
- AET_t^c = actual evapotranspiration during period t for crop c (depth units);
- $AREA^c$ = area irrigated under crop c ;
- a_t = variable that relates A_a and e_t ;
- c = crop index;
- D_t^c = average root depth of crop c in period t ;
- DP_t^c = deep percolation during the period t for crop c ;
- e_t = evaporation rate in period t ;
- F_i = fitness value;
- $f(x)$ = objective function value;
- IR_t = rainfall in period t (depth units);
- k = total number of constraints;
- ky_g^c = yield response factor for growth stage g of crop c ;
- NC = number of crops;
- NGS = number of growth stages;
- NP_g = number of time periods in growth stage g ;
- OVF_t = overflow from the reservoir during the period t ;
- PET_t^c = potential evapotranspiration during period t for crop c (depth units);
- Q_t = inflow into the reservoir during the period t ;
- R_t = release for irrigation from the reservoir for the period t ;
- RG_g^c = irrigation allocation for the growth stage g of the crop c (depth units);
- S_{max} = maximum active storage capacity;
- S_t = active storage at the beginning of the period t ;
- SM_t^c = available soil moisture at the beginning of the period t for the crop c ;
- SM_{max}^c = available soil moisture at the field capacity for the crop c ;
- x_t^c = irrigation water allocated to crop c in period t (depth units);

- X_t = total amount of irrigation water available at the farm level;
- δ_j = penalty coefficient;
- ϵ = -1 for maximization ($+1$ for minimization);
- η = conveyance efficiency; and
- $\langle \phi_j \rangle$ = amount of violation.

Subscripts

- a = active storage;
- c = crop;
- g = growth stage;
- t = time period; and
- 0 = dead storage.

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