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Hydrological Drought Forecasting through Markov Mixture Modelling

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Abstract

Canal withdrawals from a river (directly through a diversion structure, or implicitly through reservoir storage) for irrigation depend on the crop water requirements and on the available supply. The crop water demands in turn depend on the rainfall in the command area, among other things. Thus a knowledge of the availability of both streamflow and rainfall, in advance, will be helpful in taking short term decisions on canal withdrawals for irrigation. A deficiency of streamflow and or rainfall indicates a hydrological drought. An estimate or forecast of this possible deficiency in a future period will be helpful in terms of getting prepared to deal with the situation and take appropriate action. A mathematical model to predict, one time step in advance, the variation of streamflow, Q, and rainfall in the command area, R, with reference to their mean values Q_o and R_o, respectively is presented in this paper.

A Markov Mixture (MM) model is proposed to forecast the status of both streamflow and rainfall in a single state. Streamflow refers to the river flow at the site of a diversion structure, or inflow to a reservoir as the case may be; and, rainfall refers to the rainfall in the command area in either case. The MM model has the advantage that it accounts for state variations in each period and state transitions between periods, features which are not considered explicitly in other forecasting models.

Model application to the Right Bank Canal command of Malaprabha reservoir in Karnataka State is illustrated. A ten day time period is chosen because of the importance of short term decision intervals in irrigation operation.

INTRODUCTION

It is important to be able to foresee the status of water supply in future periods in irrigated agriculture to bring about a sense of preparedness to effectively plan the field operations. Decisions on canal withdrawals from a river (directly through a diversion structure, or implicitly through reservoir storage) for irrigation are made based on crop water requirements and available supply. Crop water demands in turn depend on the rainfall in the command area, among other things. The surface supply depends on the river flow. Thus a knowledge of the availability of both streamflow and rainfall, in advance, will be helpful in taking short term decisions on canal withdrawals for irrigation. A deficiency of streamflow and or rainfall indicates a hydrological drought. An estimate or forecast of this possible deficiency in a future period will be helpful in terms of getting prepared to deal with the situation and take appropriate action. A mathematical model to predict, one time step in advance, the variation of streamflow, Q, and rainfall in the command area, R, with reference to their mean values Q_o and R_o, respectively, is presented in this paper.

A Markov Mixture (MM) model is proposed in the present study to forecast the status of both streamflow and rainfall in a single state. The streamflow refers to the river flow at the site of a diversion structure, or the inflow to a reservoir as the case may be; and, rainfall refers to the rainfall in the command area. The MM model has the advantage that it accounts for state variations in each period and state transitions between periods, features which are not considered explicitly in other forecasting models. Markov Mixture models [Jackson, 1975] were developed earlier for generating streamflows to preserve the statistics of drought lengths. Vedula et al. [1991] used a Markov Mixture model to forecast streamflows. The present paper follows this later paper in developing a model for forecasting a combined state for streamflow and rainfall.

Model Formulation

It is assumed that streamflow as well as rainfall follows a first order stationary Markov process. The basic property of the Markov process is that the probability, P_{ij} of transition from state i in period t to state j in period t+1, given that the system is in state i in period t, is independent of the history of the process earlier to the time period t. In other words,

$$P(X_t/X_{t-1}, X_{t-2},...) = P(X_t/X_{t-1}) \quad (1)$$

where X_t, X_{t-1}, ... are the values of the variable X at different time steps t, t-1,...

Four possible states are considered in a given period to denote the combined status of the streamflow, Q, and rainfall in the command area, R: State 1 in which $Q<Q_o$ and $R<R_o$, State 2 in which $Q<Q_o$ and $R \geq R_o$, State 3 in which $Q \geq Q_o$ and $R<R_o$, and State 4 in which $Q \geq Q_o$ and $R \geq R_o$. For a given state of the system in the current period, the model estimates the future state in the next period by computing the conditional expectations of streamflow and rainfall values for the next period. These make use of the state transition probabilities from one period to the next and a Thomas-Fiering type generation scheme for both streamflow and rainfall.

It is assumed that the state of the system in the current period t is known and it is required to forecast the state in the period t+1. The streamflow in a river is a function of the watershed and pstream of the withdrawal site, whereas the rainfall in the irrigated area depends on the local climatic characteristics. Also the irrigated area is usually located far from the watershed which contributes to the inflow at the withdrawal site. It is assumed, therefore, that the streamflow at the withdrawal site and the rainfall in the command area are independent of each other. This may not be strictly true in all cases, however.

Because of the assumption of independence, the transition probabilities of streamflow and of rainfall can be separately estimated from historical data. Thus streamflow (or rainfall) will have two classes in each time period: one in which the value is below the mean and another in which it is above it. Let i=1 denote the class in which Q (or R) < Q_o (or R_o) and i=2 the class in which Q (or R) $\geq$ Q_o (or R_o).

The following parameters are defined in the formulation.

$P_{ij}^{t\,t+1}$ = transition probability that the state of the system will be in class j in period t+1 given that it is in state i in period t,

$f_{ij}^{t\,t+1}$ = probability of streamflow transition from class i in period t to class j in period t+1,

$r_{ij}^{t\,t+1}$ = probability of rainfall transition from class i in period t to class j in period t+1,

F_{ij} = lag one serial correlation coefficient of streamflow in class j in period t+1 with that in class i in period t,

R_{ij} = lag one serial correlation coefficient of rainfall in class j in period t+1 with that in class i in period t.

Recognising that the index i and j are always associated with the current period t and the future period t+1, respectively, they are omitted in the notation for the state transition probabilities (P), transition probabilities of streamflow (f) and rainfall (r) from here on in this paper. The suffix ij indicates a transition from class i in period t to class j in period t+1.

A Thomas-Fiering type generating scheme is used to generate the streamflow or rainfall belonging to a specific class in the future period given its class in the current period. Since streamflow and rainfall are considered independent, the generating scheme is used to determine the future period value of each of them independently. The streamflow, Q_j^{t+1}, denoted as Q_j henceforth, given Q_i^t, denoted as Q_i henceforth, is given by the expectation

$$E\{Q_j/Q_i\} = \mu_j^F + F_{ij}\,(\sigma_j^F/\sigma_i^F)\,(Q_i - \mu_i^F) \text{ for all j.} \tag{2}$$

where μ_i^F and σ_i^F are the mean and standard deviation of streamflow in period t, and values with subscript j, corresponding to the period t+1.

The estimated value of streamflow for the future period, Q^{t+1} for given i is computed as

$$Q^{t+1} \text{ (given i)} = \Sigma_j f_{ij}\, E\{Q_j/Q_i\} \tag{3}$$

In a similar way, the rainfall estimate R^{t+1} for the future period, for given i, is computed as

$$R^{t+1} \text{ (given i)} = \Sigma_j r_{ij}\, E\{R_j/R_i\} \tag{4}$$

where

$$E\{R_j/R_i\} = \mu_j^R + R_{ij}\,(\sigma_j^R/\sigma_i^R)\,(R_i - \mu_i^R) \text{ for all j,} \tag{5}$$

in which, μ_i^R and σ_i^R are the mean and standard deviation of rainfall in period t, and values with j corresponding to the period t+1.

For the classes of streamflow and rainfall in the current period, estimated values of Q^{t+1} and R^{t+1} for the future period, t+1, are thus determined. These two values then define the class in which the system lies in period t+1.

Application

The model is applied to the Right Bank Canal command area of the Malaprabha reservoir project in Karnataka State, India. A total of 36 periods are considered in a year with 3 periods in each month. The first two periods in a month are of ten days duration and the balance comprises of the third. A ten day time interval is chosen keeping in view the value of short term forecsts of the state of the system in irrigation operation.

Thirty five years (1951-52 to 1985-86) of concurrent streamflow and rainfall data were used in the study. Spatially averaged Theissen weighted rainfalls for the whole of the irrigated area in each period are used.

Table - 1 : Mean Streamflow and Rainfall (May 1951 to June 1986)

Period	Streamflow M m^3mm	Rainfall
10	55.51	57.31
11	48.31	54.96

First the mean values of streamflow and rainfall are determined for each period. Table 1 gives the mean values of streamflow and rainfall for periods t=10 and 11, for use in the example problem discussed later in the paper. The transition probabilities of the combined state of streamflow and rainfall are then determined from concurrent data of 35 years, for all the four possible states, using a relative frequency approach, for each period and tabulated.

Table - 2 : State Transition Probabilities for period t=10

i	j: 1	2	3	4
1	0.3333	0.3333	0.1111	0.2222
2	0.5000	0.2500	0.1250	0.1250
3	0.4167	0.0833	0.2500	0.2500
4	0.1667	0.1667	0.3333	0.3333

Table 2 shows these for a typical period t=10. Two classes are defined for each period into which that period's streamflow (or rainfall) falls: class 1 in which values are below the mean and class 2 in which they are above the mean. For each period, the means and the standard deviations of streamflows (and rainfalls, separately) in each of the two classes are determined. Also the transition probabilities of streamflow (and rainfall, separately) from each class in a given period to each class in the next period are estimated using a relative frequency approach, for all periods.

Table 3. Statistics of Streamflow and Rainfall for period t=10

class i	Class interval	Mean	Std. Dev.	Trans. Prob. j=1	Trans. Prob. j=2	Corr. Coeff. j=1	Corr. Coeff. j=2
			Streamflow (M m³)				
1	12.3- 55.5	34.58	11.13	0.7059	0.2941	0.5729	-0.3872
2	55.5-122.3	75.27	17.71	0.4444	0.5556	-0.1944	0.0203
			Rainfall (mm)				
1	2.40 - 57.31	30.61	18.03	0.5714	0.4286	-0.2302	0.2332
2	57.31-199.90	97.36	40.94	0.5714	0.4286	-0.4732	0.0877

Table3 shows the class intervals, means, standard deviations and the transition probabilities for streamflow and rainfall separately for period t=10. Lag one serial correlation coefficient of streamflow of a given class in a given period to that in any of the classes in the next period is estimated, for all periods. Similarly the lag one serial correlation coefficients for rainfall are also estimated. Table 3 also shows the correlation coefficients for a typical period, t=10.

Example

ExampleThe following example shows how to predict the state of the system in period 11 given the state in period 10. Let the streamflow and rainfall in period 10 be 42 M m^3 and 75 mm respectively. As the streamflow is less than its mean of 55.51, and the rainfall is higher than its mean of 57.31 for this period, the state of the system in period 10 is 2.

It is noted from Table 3 that, for the period t=10, the streamflow is in its class 1, and rainfall in its class 2.

The expected value of streamflow in period 11, given that it is 42 (class 1) in period 10, is given by Eq. 2 using Table 3 and μ_1^{11}=31.33, μ_2^{11}=70.94, σ_1^{11}=11.81, σ_2^{11}=27.81 as:

$E\{Q_j/Q_1\}$ = 31.33+0.5729(11.81/11.13)(42-34.58)=35.84 for j=1with probability of f_{11}=0.7059, in class 1, and

$E\{Q_j/Q_1\}$ = 70.94-0.3872(27.81/11.13)(42-34.58)=63.76 for j=2 with a probability of f_{12}=0.2941, in class 2.

Therefore, the estimated value of $Q^{t=11}$ =35.84(0.7059)+63.76(0.2941)=44.05Similarly, the expected value of rainfall, using Eq. 5 and the Table 3 works out to 34.19 for class 1, in period 11, with a probability of 0.5714, and 86.81 in class 2 with a probability of 0.4286. This gives an estimated value of $R^{t=11}$ of 56.74 in period 11.

In period 11, the estimated Q lies in class 1 ($Q<Q_0$) and rainfall in class 2 (R_R_0). This corresponds to state 2 of the system. Thus, given the state of the overall system in the current period t=10 to be 2, the state of the overall system in the future period t=11 is predicted to be 2.

State Transition Probabilities

It is to be noted that the state transition probabilities, P_{ij}, in Table 2 are not used in the computations. These are not necessary since the transition probabilities of streamflow and rainfall have already been estimated. However, P_{ij} values can also be computed from the streamflow and rainfall transition probabilities because of the assumption of independence. In the present example, the transition probability, P_{23}, can be computed as the product of f_{12} and r_{21}, which is 0.2941*0.5714=0.1680, as against the value, 0.1250, directly estimated from the historical data, in Table 2. The difference is attributed to the errors in the assumption of independence and of the first order Markov property, in addition to the bias in estimating the parameters in the model, due to limited sample size.

References

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